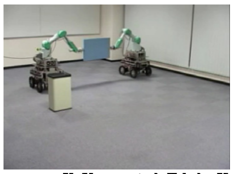
Control of the Physical Interaction in/with Multi-Robot Systems

Dongjun Lee
Interactive & Networked Robotics Laboratory
Department of Mechanical & Aerospace Engineering
Seoul National University

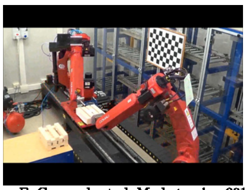
Multi-Robot Physical Interaction


Kube et al, U. Alberta

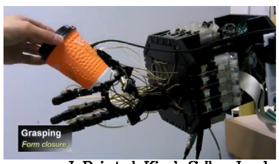

K. Kosuge et al, Tohoku U.


D. Floreano et al, EPFL


D. Rus et al, CMU


F. Caccavale et al, Mechatronics, 2013

V. Kumar et al, UPenn

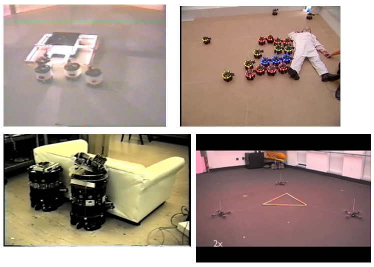


J. Dai et al, King's College London

Categorization

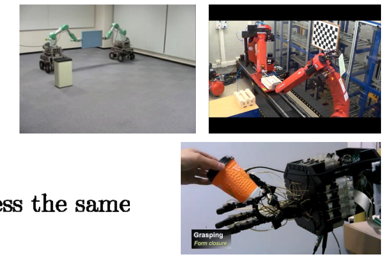
Decentralized

- Local control and communication
- Behavioral control, swarming, etc.
- Large number of robots
- Loose mechanical coherency



Centralized

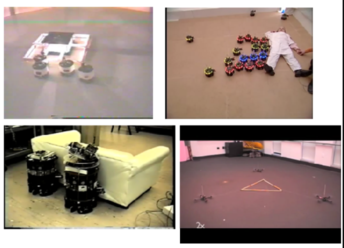
- Central control and communication
- Precise mechanical coherency
- Relatively small number of robots
- Single robot or multiple robots more or less the same



Issues of Their Own

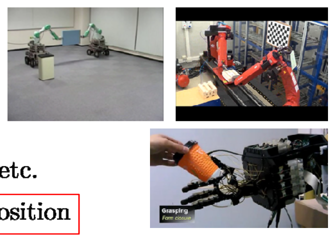
Decentralized

- Desired behavior only from local interaction
- Partial connectivity and latency
- Collision avoidance vs separation prevention
- Provable emergent behavior
- Consensus/synchronization on graph



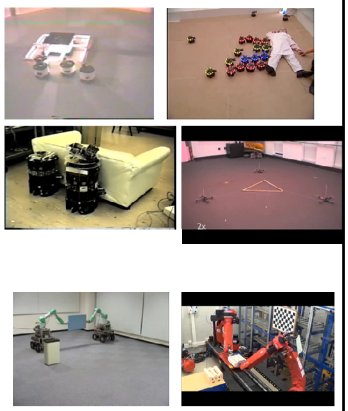
Centralized

- Central communication and control expensive
- Precise and tight coordination
- High performance from real robots
- Real robots w/ complex dynamics, constraints, etc.
- Hybrid position/force control, behavior decomposition



Content

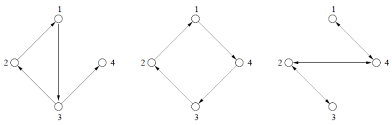
- Decentralized vs Centralized
- Decentralized control of physical interaction
  - Overview
  - Multi-UAV teleoperation
  - Multi-user haptic interaction
- Centralized control of physical interaction
  - Overview
  - Multiple mobile manipulators
  - Multiple quadrotor-manipulator systems
  - Spherically-connected multi-quadrotor (SmQT) systems
- Conclusion and future directions



Decentralized Physical Interaction Control

- Simple first-order consensus equation:

$$\boxed{\dot{x}_i = -\sum_{j \in \mathcal{N}_i} w_{ij}(x_i - x_j)}$$

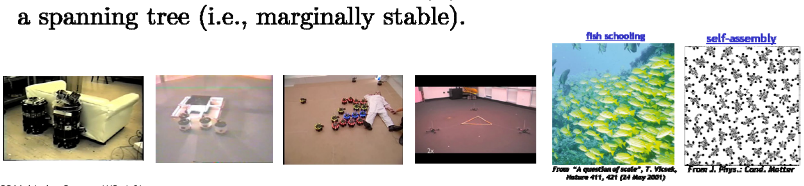


where $\mathcal{N}_i \in \mathcal{V}$ is information neighbor on graph $G = (\mathcal{V}, \mathcal{E}, \mathcal{W})$.

- Each robot abstracted by a simple dynamics (e.g., point mass in $\Re^3$)
- System communication/control restricted by the topology of graph G
- Closed-loop dynamics:

$$\boxed{\dot{x} = -Lx, \quad x = [x_1, x_2, ..., x_n] \in \Re^{3n}}$$

where L is Laplacian matrix with $\lambda_i(L) \geq 0$ with 0 being simple iff G has a spanning tree (i.e., marginally stable).

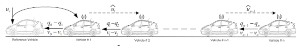


Decentralized Physical Interaction Control

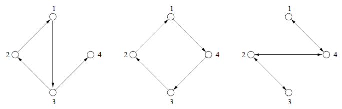
- Total control law = collective control + local control

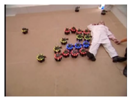
$$\boxed{\dot{x}_i = u_i(t) - \sum_{j \in \mathcal{N}_i} w_{ij}(x_i - x_j) \quad \Rightarrow \quad \dot{x} = -Lx + u}$$

- $u_i(t) \in \Re^3$: collective control for some agents to drive whole group (e.g., human command, virtual leader)
- $w_{ij}(||x_i - x_j||)$: local control to maintain coherency while avoiding collision on graph G

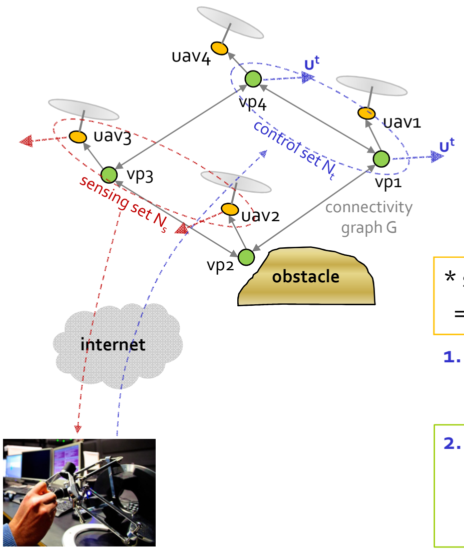


- How to maintain local behavior (e.g., avoidance, coherency) even with unpredictable $u_i(t)$? (cf. string stability, Swaroop et al, TAC96)
- Collective control $u_i(t)$ often cognitive, whereas local control $w_{ij}(||x_i - x_j||)$ typically mechanical
- **Pseudo physical interaction** $\Leftarrow$ indirectly via motion control





Multi-UAV Teleoperation

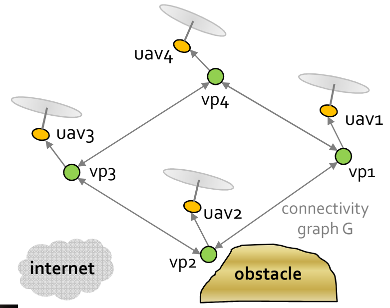


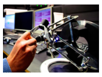
- issues/challenges:
  - single user can manage only small-DOF
  - information-flow among UAVs should be distributed, yet, no collision/separation under arbitrary human tele-command

* semi-autonomous teleoperation
= teleoperation + local autonomous control

1. UAV control layer (bakcstepping):
   - under-actuated UAV tracks its own kinematic virtual point (VP)
2. VP control layer:
   - *N* VPs as a deformable flying object on G
   - deforms to obstacles w/o VP-VP separation or VP-obstacle/VP-VP collisions
3. teleoperation layer:
   - PSPM for flexible/stable teleoperation

Distributed VP Control Layer





- render N kinematic VPs as a N-nodes deformable flying object with artificial potentials distributed over connectivity graph G

- same architecture can be used for interaction with real objects

kinematic VP

$$\dot{p}_i(t) := u_i^t + u_i^c + u_i^o$$

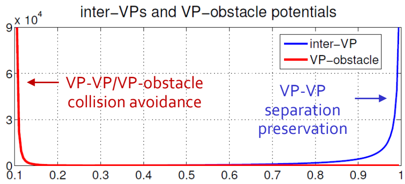
VP velocity; teleoperation command; VP-VP potential; VP-obstacle potential

$$u_i^c := -\sum_{j\in\mathcal{N}_i} \frac{\partial \varphi_{ij}^c(\|p_i - p_j\|^2)^T}{\partial p_i}$$

$j\in\mathcal{N}_i$: neighbors on connectivity graph G

$$u_i^o := -\sum_{r\in\mathcal{O}_i} \frac{\partial \varphi_{ir}^o(\|p_i - p_r^o\|)^T}{\partial p_i}$$

$r\in\mathcal{O}_i$: obstacle set



distance-based artificial potential
→ rotational symmetry

Swarming Property [TMech13]

$$\dot{p}_i(t) := u_i^t + u_i^c + u_i^o \qquad V(t) := \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\varphi_{ij}^c(\|p_i - p_j\|) + \sum_{i=1}^{N}\sum_{r\in\mathcal{O}_i}\varphi_{ir}^o(\|p_i - p_r^o\|)$$

collision/separation impending

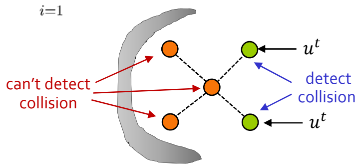
Prop. 1: Suppose $\|u_i^t\| \leq \bar{u}\ \forall t \geq 0$, and, if $V(t) \geq M$, $\exists$ at least one VP, s.t.,

$$\left\| \sum_{j\in\mathcal{N}_s}\frac{\partial\varphi_{sj}^c}{\partial p_s} + \sum_{r\in\mathcal{O}_s}\frac{\partial\varphi_{sr}^o}{\partial p_s}\right\| \geq \frac{\sqrt{N_t}+\delta_{st}}{2}\bar{u} \qquad \begin{aligned}\delta_{st} &= 1 \text{ if } s\in Nt;\\ \delta_{st} &= 0 \text{ if } s\notin Nt\end{aligned}$$

Then, all VPs are stable (i.e., bounded $\dot{p}_i$); no VP-VP/VP-obstacle collisions; and no VP-VP separations.

$$\frac{dV}{dt} = \sum_{i=1}^{N}\underbrace{\left(\sum_{j\in\mathcal{N}_i}\frac{\partial\varphi_{ij}^c}{\partial p_i} + \sum_{r\in\mathcal{O}_i}\frac{\partial\varphi_{ir}^o}{\partial p_i}\right)}_{=:W_i^T}\dot{p}_i = \sum_{i=1}^{N}W_i^T(-W_i + u_i^t) \leq \sum_{i=1}^{N}(-\|W_i\|^2 + \delta_{it}\bar{u}\|W_i\|)$$

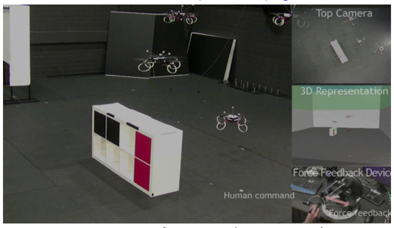
$$\leq -\left(\|W_s\| - \frac{\delta_{st}\bar{u}}{2}\right)^2 + N_t\frac{\bar{u}^2}{4} \leq 0 \quad \rightarrow V(t) \leq M$$



- only *one* VP needs to detect $V(t) \geq M$, w/ potential not exactly aligned
- stable for *any* bounded teleoperation command u_i^t $\Leftarrow$ guaranteed by PSPM

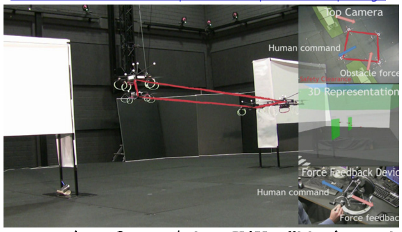
Experiments

distributed multi-UAV teleoperation: flying-over obstacle



∗ flying-over obstable w/o inter-UAV collision/separation on graph G under arbitrary human command

distributed multi-UAV teleoperation: adapt to narrow passage



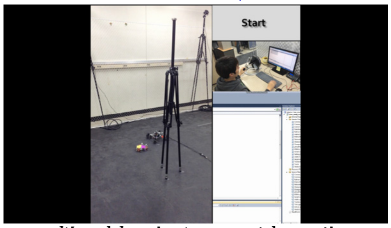
∗ squeeze/reconfigure w/o inter-UAV collision/separation on graph G under arbitrary human command

* joint work with Max Planck Institute for Biological Cybernetics, Tübingen, Germany

haptic teleoperation of UAV

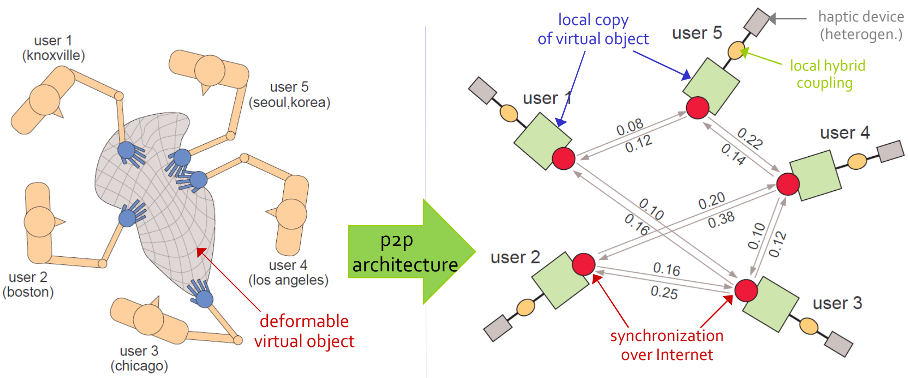
∗ open cap using UAV teleoperation with haptic feedback and communication delay

multi-modal semi-autonomous teleoperation of UAV/UGV



∗ multi-modal semi-autonomous teleoperation crucial for complex real-world applications

Multiuser Haptic Interaction



- Multiple haptic devices (i.e., robots) interconnected over graph G
- Each robot has "real" physical interaction with human
- Source of physical instaility: human-device interaction, information delay
- Consensus for consistency (i.e., coordination) and passivity for stability

Discrete-Time Passivity

Suppose VO synchronization gains B_i, K_i are set to be:

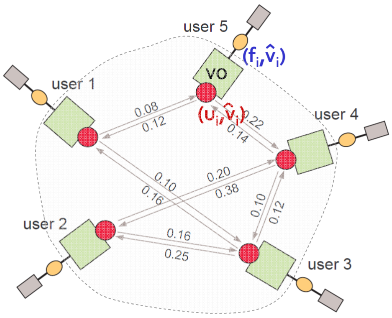
$$B_i \geq \sum_{j \in \mathcal{N}_i} \frac{N_{ij} + N_{ji}}{2} \max_k [T_j(k)] K_{ij} \qquad \forall \text{ users } i=1,\dots,N$$

Then, total p2p architecture is **N-port discrete-time passive**: $\forall \bar{M} \geq 0$,

$$\sum_{k=0}^{\bar{M}} \sum_{i=1}^{N} \hat{v}_i^T(k) f_i(k) T_i(k) \geq V(\bar{M}+1) - V(0) + \sum_{k=0}^{\bar{M}} \sum_{i=1}^{N} ||\hat{v}_i(k)||_B^2 T_i(k)$$

total energy
= VO kinetic (M) + VO potential (K)
+ synchronization potential (K_{ij})

VO damping (B)



- non-iterative passive integrator [DSCC08]
  + passive synchronization [ACC10]
- extend [Lee&SpongTRO06] to discrete domain
- robust stability for *any* devices & passive users
- not require specific kind/number of device/user
  → portability/scalability for heterogeneous devices/users

Local Copy Synchronization

If user forces $f_i(k)=0$ and VO damping B is positive-definite, $v_i(k) \rightarrow 0$ and VO local copies will be **configuration-synchronized** s.t.

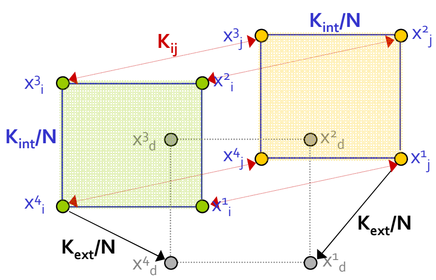
$$\left[\mathcal{P} + I_{N\times N} \otimes \frac{K}{N} \right] (x(k) - 1_N \otimes x_d) \rightarrow 0$$

effect of synchronization K_{ij}

effect of VO spring K

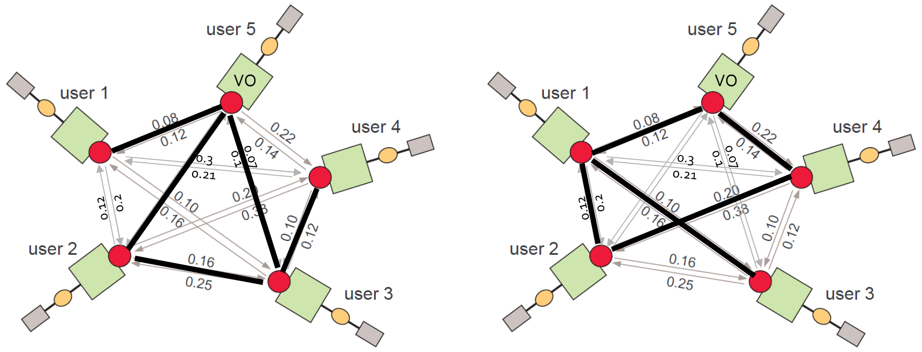
Kronecker product

$$P_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i} K_{ij} & i = j \\ -K_{ij} & i \neq j \text{ and } e_{ij} \in \mathcal{E} \\ 0_{3n\times 3n} & \text{otherwise} \end{cases}$$



- all the VO local copies' configurations
  $x(k) = [x_1(k); x_2(k); \dots x_N(k)] \in \mathbf{R}^{3nN}$
  converge to stable equilibria
  $x(k) \rightarrow \text{null}(P) \cap \text{null}(I_N \otimes K)$
- VO synchronization guaranteed
  with $\text{null}(P) = \{x_i = x_j = d, d \in R^{3n}\}$
  if **G** is *connected* [Huang&LeeACC10]
- K_{int} : VO internal shape
- K_{ext}: symmetry breaking
  e.g. if $K_{ext} = 0$, $x_i \rightarrow x_j \rightarrow I_N \otimes c$, $c \in R^3$

Network Topology Optimization



Which (connected) network topology should we choose?

→ ***fastest mixing graph*** G_{op} **from the set of all candidates graphs** G_i

information mixing model

$$p_i(k+1) = \left(\sum_{j \in \mathcal{N}_i} K_{ij} \right)^{-1} \sum_{j \in \mathcal{N}_i} K_{ij} p_j(k+1-N_{ij})$$

user i's information state

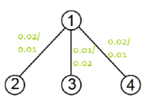
normalization

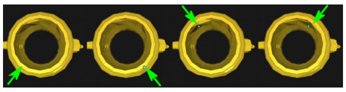
K_{ij}: information mixing strength

communication delay

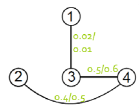
Experiments

best topology



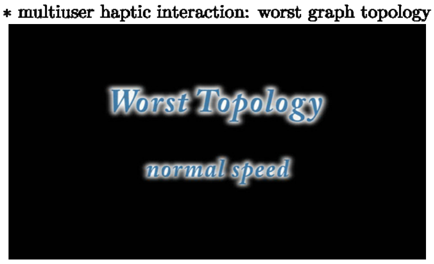


worst topology

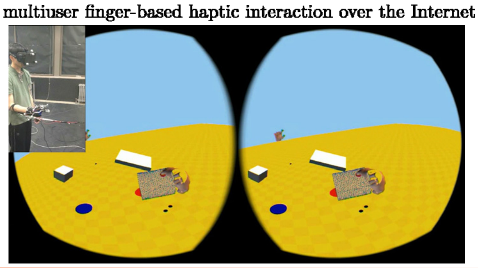


∗ multiuser haptic interaction: best graph topology

∗ multiuser haptic interaction: worst graph topology



∗ multiuser finger-based haptic interaction over the Internet



Centralized Physical Interaction Control

- Single robot as kinematic or dynamic system:

$$\boxed{\dot{x}_i = J_i(q_i)\dot{q}_i, \quad M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i = \tau_i + f_i}$$

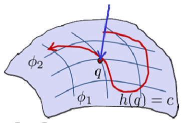
- If stack-up, multirobot system the same as single robot system:

$$\boxed{\dot{x} = J(q)\dot{q}, \quad M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau + f}$$

w/ $x = [x_1, x_2, ..., x_n]$, $q = [q_1, q_2, ..., q_n]$, $\tau = [\tau_1, \tau_2, ..., \tau_n]$, $f = [f_1, f_2, ..., f_n]$, $J = \text{diag}[J_1, J_2, ..., J_n]$, $M = \text{diag}[M_1, M_2, ..., M_n]$, $C = \text{diag}[C_1, C_2, ..., C_n]$

- **Null-space based control:** with two tasks $r_1 = f_1(q)$, $r_2 = f_2(q)$, r_1 of higher priority,

$$\boxed{\dot{q} = J_1^+\dot{r}_1 + (J_2P_1)^+[\dot{r}_2 - J_2\dot{q}_1]}$$



where P_1 is projection to null-space of J_1.

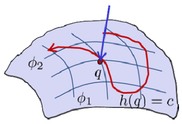
- **Hybrid position/force control:** with the task specified by holonomic constraint $h(q) = c$ (or $q = f(\phi)$, $\dot{q} = J(\phi)\dot{\phi}$),

$$\boxed{\begin{aligned}\tau = M(q)J(\phi)(\ddot{\phi}_d - K_v\dot{e}_\phi - K_pe_\phi) + [C(q,\dot{q})J(\phi) + M(q)\dot{J}(\phi)]\dot{\phi} \\ + g(q) - f + A^T(q)[\lambda_d - K_f\int(\lambda - \lambda_d)dt]\end{aligned}}$$

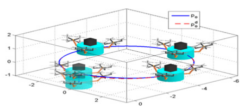
Centralized Physical Interaction Control

- Kinematic or dynamic modeling of multi-robot system:

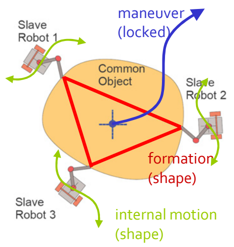
$$\boxed{\dot{x} = J(q)\dot{q}, \quad M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \tau + f}$$



- Null-space control, hybrid control $\Rightarrow$ behavior decomposition
- Multi-robot with centralized control $\Rightarrow$ more or less same as single robot
- **Then, what is so unique about multi-robot system?**
  - Large-DOF $\Rightarrow$ difficult to control all the same
  - Large-DOF $\Rightarrow$ richer behavioral decomposition possible/demanded
  - As compared to a single robot, real multi-robot would likely have
    - More complex dynamics (e.g., platform-arm system)
    - More abundance of constraints (e.g., wheels, under-actuation)
    - Heterogeneity among the robots
- **Behavior decomposition** w/ complex dynamics, constraint, heterogeneity

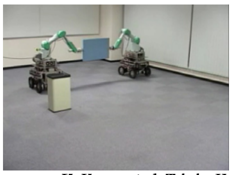


Multirobot Fixture-Less Grasping



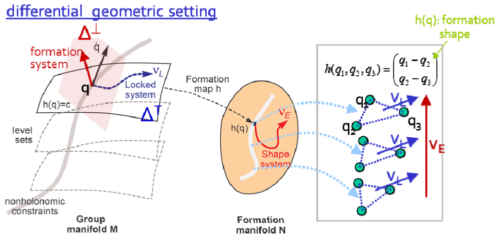
- total-DOF = 15
- three behaviors:
  1) grasping
  2) grasped object maneuver
  3) internal motion (e.g., avoidance, reconfiguration)

→ decomposition into these three behaviors even with nonholonomic constraints?



K. Kosuge et al, Tohoku U.

differential geometric setting



orthogonal decomposition w.r.t. M(q)-metric

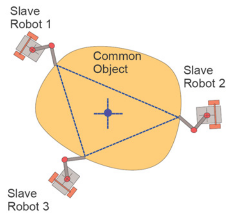
$$\mathcal{D}^\top = (\mathcal{D}^\top \cap \Delta^\top) \oplus (\mathcal{D}^\top \cap \Delta^\perp) \oplus \mathcal{D}^c$$

permissible motion | permissible maneuver+internal | permissible grasping | quotient: perturb maneuver & formation

INRoL

Behavior Decomposition and Control

* hierarchical control
  = simultaneous/separate control of each behavioral mode autonomously or teleoperatedly

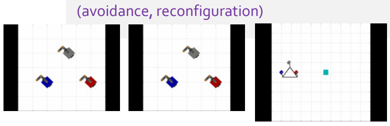


$$\dot q = \lambda_h \xi^h + (u_x \xi^x + u_y \xi^y) + \sum_{i=1}^{3} \alpha_i \xi_i^r + \sum_{i=1}^{3} \beta_i \xi_i^t + \sum_{k=1}^{3} 0 \cdot \xi_k^c$$

grasping | grasping

internal motion (avoidance, reconfiguration)

object maneuvering



simple autonomous

autonomous, yet, rich (or teleoperation?)

cognitive

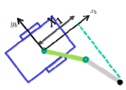
required level of intelligence

dynamics of ϕ under (ξ_x, ξ_y)-modes: with $u_x = 1, u_y = 0$

$$\dot\phi_i = -\frac{1}{X_1}\sin\phi_i$$

stability depends on X_1

equilibrium: $\phi = 0, \pi$
(1) if $X_1 > 0$, $\phi \to 0$
(2) if $X_1 < 0$, $\phi \to \pi$



INRoL

NPD Expression

grasp regulation $\lambda_h = -S_E(\frac{\partial \varphi}{\partial h})$

no-excitation

$$\dot{q} = \lambda_h \xi^h + (u_x \xi^x + u_y \xi^y) + \sum_{i=1}^{3} \alpha_i \xi_i^r + \sum_{i=1}^{3} \beta_i \xi_i^t + \sum_{k=1}^{3} 0 \cdot \xi_k^c$$

$(D^\top \cap \Delta^\perp)$ grasping: simple autonomous

object maneuvering: teleoperation

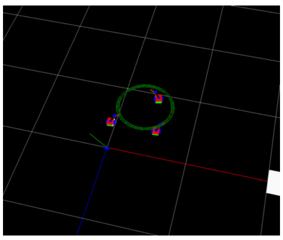
internal motion: autonomous (rich)

grasping(D^c): simple autonomous

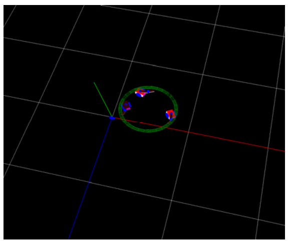
$$\begin{bmatrix}
0 & 0 & 0 & -l_1 l_2 \cos\phi_1 \sin\theta_2^1 & 0 & 0 & \cos\phi_1 \frac{x_1}{X_1} & \cos\phi_1 \frac{y_1}{X_1} \\
0 & 0 & 0 & -l_1 l_2 \sin\phi_1 \sin\theta_2^1 & 0 & 0 & \sin\phi_1 \frac{x_1}{X_1} & \sin\phi_1 \frac{y_1}{X_1} \\
1 & 0 & 0 & 0 & 0 & 0 & -\sin\phi_1 \frac{1}{X_1} & \cos\phi_1 \frac{1}{X_1} \\
-1 & 0 & 0 & l_2 \cos(\theta_1^1 + \theta_2^1) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -X_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -l_1 l_2 \cos\phi_2 \sin\theta_2^2 & 0 & \cos\phi_2 \frac{x_2}{X_2} & \cos\phi_2 \frac{y_2}{X_2} \\
0 & 0 & 0 & 0 & -l_1 l_2 \cos\phi_2 \sin\theta_2^2 & 0 & \sin\phi_2 \frac{x_2}{X_2} & \sin\phi_2 \frac{y_2}{X_2} \\
0 & 1 & 0 & 0 & 0 & 0 & -\sin\phi_2 \frac{1}{X_2} & \cos\phi_2 \frac{1}{X_2} \\
0 & -1 & 0 & 0 & l_2 \cos(\theta_1^2 + \theta_2^2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -X_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -l_1 l_2 \cos\phi_3 \sin\theta_2^3 & \cos\phi_3 \frac{x_3}{X_3} & \cos\phi_3 \frac{y_3}{X_3} \\
0 & 0 & 0 & 0 & 0 & -l_1 l_2 \cos\phi_3 \sin\theta_2^3 & \sin\phi_3 \frac{x_3}{X_3} & \sin\phi_3 \frac{y_3}{X_3} \\
0 & 0 & 1 & 0 & 0 & 0 & -\sin\phi_3 \frac{1}{X_3} & \cos\phi_3 \frac{1}{X_3} \\
0 & 0 & -1 & 0 & 0 & l_2 \cos(\theta_1^3 + \theta_2^3) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -X_3 & 0 & 0
\end{bmatrix}$$

ξ_i^r $\quad$ ξ_i^t $\quad$ (ξ_x, ξ_y)

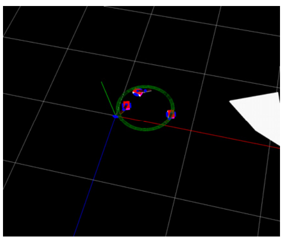
Simulation



- autonomous obstacle avoidance using combination of maneuver mode and internal dynamics mode
- rigid grasping enforced with **no** grip-holding fixture



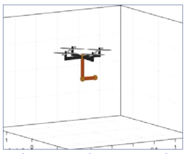
- autonomous grasping control
- object maneuver haptic teleoperation completely decoupled from grasping behaviors



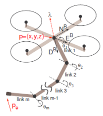
- object teleoperation with interaction force feedback
- rigid grasping maintained regardless of object interaction

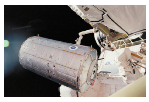
Quadrotor-Manipulator System

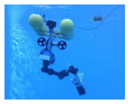
- Dexterous aerial manipulation using quadrotor-arm system is promising.
- Quadrotor-arm dynamic coupling → dynamics formulation necessary.
- QM system dynamics very complex with 2-DOF under-actuation.
- Slow/imprecise quadrotor flying + fast/precise manipulator control.
- Reveal fundamental underlying dynamics structure; exploit it for control.

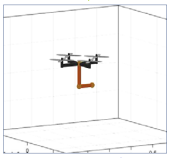

quadrotor-arm dynamic coupling

quadrotor-arm dynamic coupling








quadrotor-arm coarse-fine control

$$M(r)\ddot q + C(r,\dot r)\dot q + g(q) = \tau + f$$

$$m_L \ddot p_L + g_L = \tau_L = \lambda R_o(\phi)$$
$$M_E(r)\ddot r + C_E(r,\dot r)\dot r = \tau_E$$

center-of-mass dynamics in E(3) (coarse operation)

platform rotation + internal motion (precise operation)

applicable to **any** vehicle-manipulator systems

Configuration-Space Decomposition

tangent space decomposition

$q := [p;\phi;\theta] \in \Re^n \qquad r := [\phi;\theta] \in \Re^{m+3}$

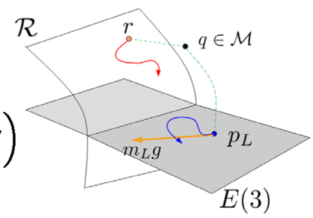
➡ Choose $h(q) := r$

➡ $\dot q = [\ \Delta_\top \quad \Delta_\perp\ ]\,\nu = \begin{bmatrix} I_3 & -\frac{1}{m_L}M_{pr}(r) \\ 0 & I_{n-3}\end{bmatrix}\begin{pmatrix}\dot p_L \\ \dot r\end{pmatrix}$

tangential distribution: integrable

orthogonal w.r.t. M(r)-metric

normal distribution: also integrable!



passive decomposition [ICRA14]

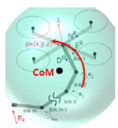
$$M(r)\ddot q + C(r,\dot r)\dot q + g(q) = \tau + f$$

$$m_L \ddot p_L + g_L = \tau_L = \lambda R_o(\phi)$$
$$M_E(r)\ddot r + C_E(r,\dot r)\dot r = \tau_E$$

center-of-mass dynamics in E(3)

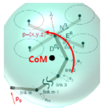
platform rotation + internal motion

- QM dynamics = centroid p_L-dynamics + internal rotation r-dynamics.
- p_L-dynamics = standard under-actuated quadrotor dynamics.
- r-dynamics = standard fully-actuated robot-arm dynamics.
- No inertial/gravity coupling w/ gravity only in p_L-dynamics.



⇒ Composed of completely-decoupled p_L and r dynamics on their manifolds.
⇒ Generalize rigid-body dynamics in SE(3) to floating multi-link systems.
⇒ Applicable to **any** vehicle-manipulator systems (e.g., ROV, space robot).

Coarse-Fine QM-System Control



- Backstepping end-effector tracking control with redundancy

$$\tau_L(\lambda,\phi)+m_L B(r) M_E^{-1}(r)\tau_E = -\gamma e_p - \alpha e_L + g_L + \eta(r) + m_L[\ddot{p}_d - \lambda \dot{e}_p - \frac{dB}{dt}\dot{r}]$$

control redundancy: cooperative control

- Slow p_L-dynamics w/ under-actuated control $\tau_L^d(\lambda,\phi_d) \Rightarrow$ **coarse control**

$$\tau_L^d(\lambda,\phi_d) := \textbf{LPF}\left[-\gamma e_p - \alpha e_L + g_L + \eta(r) + m_L[\ddot{p}_e^d - k\dot{e}_p - \tfrac{dB}{dt}\dot{r}]\right] - m_L\zeta(r)$$

freely-assignable w/o affecting control objective

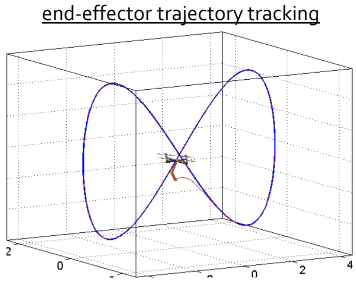
- Fast r-dynamics w/ fully-actuated control $\tau_E \Rightarrow$ **fine control**

$$m_L B M_E^{-1}\tau_E = -\gamma e_p - \alpha e_L + g_L + \eta(r) + m_L[\ddot{p}_e^d - k\dot{e}_p - \tfrac{dB}{dt}\dot{r}] - \tau_L(\lambda,\phi)$$

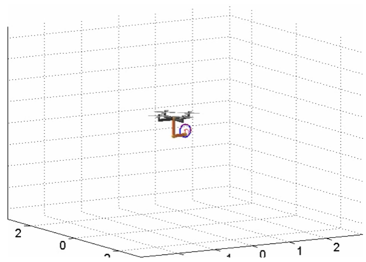
- Trajectory tracking $(e_p, e_L) \to 0$ guaranteed even with $\tau_L \neq \tau_L^d$.
- Latitude for τ_L^d: **arm sub-task task** $\zeta(r) \Rightarrow m_L B\ddot{r} \to \zeta(r)$ (e.g., impedance).
- **Singularity avoidance** by adapting cut-off frequency $w_c(\sigma(\theta))$ of LPF.
- Redundant manipulator control $\tau_E \in \texttt{nullspace}(BM_E^{-1})$:
  - Quadrotor attitude control to align $\tau_L \to \tau_L^d$. possible with kinematically decoupled manipulator
  - Optimal arm posture, collision/obstacle avoidance.

QM System Control Examples

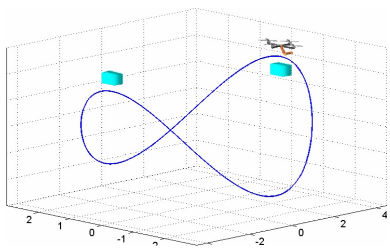
end-effector trajectory tracking



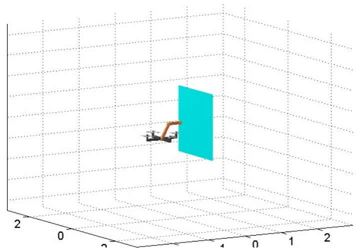
coarse-fine control



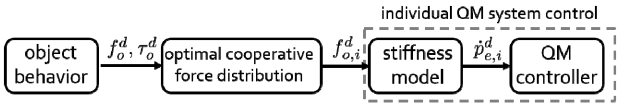
tracking+obstacle avoidance



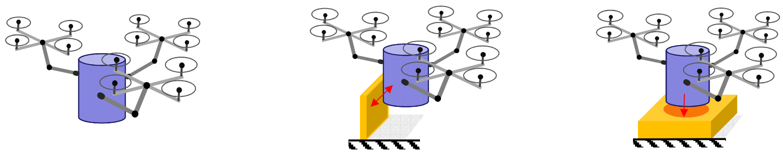
admittance force control



Hierarchical Cooperative Control Framework

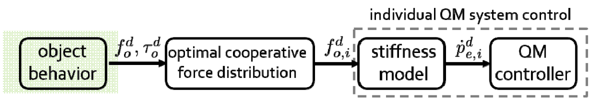


- **Object behavior design**: computes required object wrench to produce user-specific target behaviors (e.g., trajectory tracking, compliant interaction, etc.)
- **Optimal cooperative force distribution**: optimally assign contact force for each QM system to achieve the target behavior under uni-lateral/friction contact constraint
- **Individual QM system control**: admittance force control with unknown object stiffness with coarse-fine redundancy resolution



trajectory tracking — physical interaction — peg-in-hole

Object Behavior Design Layer



- **Object behavior design**: compute desired object wrench to produce user-specific behaviors (e.g., trajectory tracking, compliant interaction).
- Object rigid-body dynamics:

translation dynamics

$$m_o \ddot{p}_o + m_o g = f_o + f_{ext}$$

$$I_o \dot{w}_o + w_o \times I_o w_o = \tau_o + \tau_{ext}$$

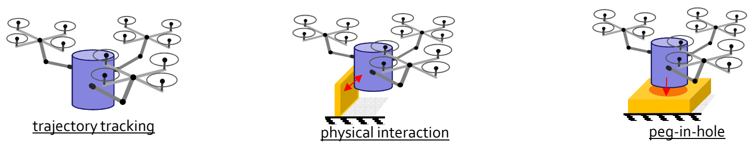
rotation dynamics

external moment

- Compute object wrench $F_o = [f_o, \tau_o]$, e.g., for impedance control:

$$f_o = m_o g + m_o \ddot{p}_o^d + D(\dot{p}_o^d - \dot{p}_o + K(p_o^d - p_o))$$

$$\tau_o = -\gamma w_o - \sigma [R_o - R_o^T]^\vee$$



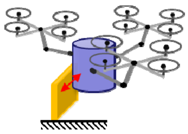
trajectory tracking — physical interaction — peg-in-hole

Optimal Cooperative Force Distribution Layer

individual QM system control

object behavior $\xrightarrow{f_o^d, \tau_o^d}$ optimal cooperative force distribution $\xrightarrow{f_{o,i}^d}$ stiffness model $\xrightarrow{\dot{p}_{e,i}^d}$ QM controller

- **Optimal cooperative force distribution**: optimally assign contact force of each QM system to achieve the target behavior w/o dropping the object (i.e., friction cone constraint).



- Jacobian relation:

$$F_o = J_o \bar{f}_o, \qquad J_o \in \Re^{6\times 3N}$$

desired object wrench
force of all N QM systems

where $\bar{f}_o = [f_{o,1}; f_{o,2}; \dots f_{o,N}] \in \Re^{3N}$ is all N QM systems' EF forces.

- J_o fat matrix with friction cone constraints $\Rightarrow$ constrained optimization:

$$\min_{f_s, f_n} \quad \alpha_1 f_s^T f_s + \alpha_2 f_n^T f_n$$

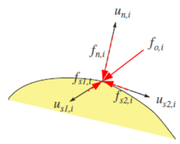
minimize internal force

$$\text{subj. to} \quad F_o = J_o \mathcal{N} f_n + J_o \mathcal{T} f_s$$

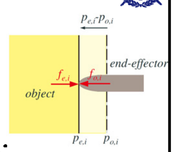
normal/tangential decomposition

$$\sqrt{f_{s1,i}^2 + f_{s2,i}^2} \geq \mu f_{n,i}, \; i = 1, 2, \cdots N$$

friction cone constraint



Admittance Force Control Layer



- Imprecise position control of quadrotor $\Rightarrow$ force control.
- Robot-arm likely not backdrivable $\Rightarrow$ admittance control.
- Local object deformation model with **unknown** stiffness matrix K_o:

$$f_{e,i} = -K_o(p_{e,i} - p_{o,i})$$

unknown stiffness

- Taget position evolution: $\dot{p}_{e,i}^d := k_1(f_{e,i} - f_{e,i}^d) + k_2 \int (f_{e,i} - f_{e,i}^d)dt + \dot{p}_{o,i}$
- Lyapunov function for admittance-like force control

$$V := \frac{1}{2} e_f^T e_f + \frac{1}{2} e_{fI}^T k_2 K_o e_{fI} + \epsilon e_f^T e_{fI} + \frac{1}{2\gamma} \dot{e}_p^T K_o \dot{e}_p$$

error in $f_{e,i}^d$-tracking
integral error $e_{fI} := \int e_f \, dt + \dot{f}_d$
error in p_e^d-tracking

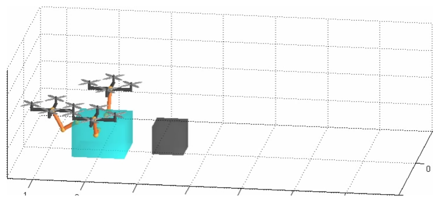
- Control generation equation

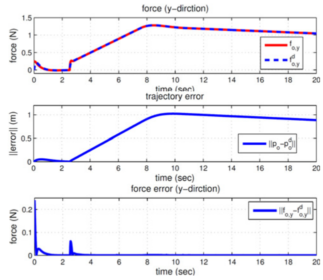
$$\boxed{\begin{aligned} \tau_L(\lambda, \phi) + m_L B(r) M_E^{-1}(r) \tau_E = & - f_L - m_L B M_E^{-1} f_E + g_L \\ & + \eta(r) + m_L[\ddot{p}_d - \beta \dot{e}_p - \gamma(e_f + \epsilon \int e_f dt) - \frac{dB}{dt}\dot{r}] \end{aligned}}$$

  - (e_p, e_f, e_{fI}) **ultimately bounded** even w/ unknown K_o & $\tau_L \neq \tau_L^d$.
  - Slower-quadrotor/faster-manipulator can be incorporated.
  - Force sensing or estimator for force feedback.

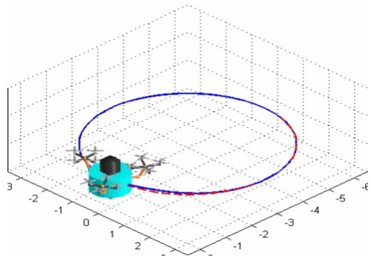
Multiple Cooperative QM-System

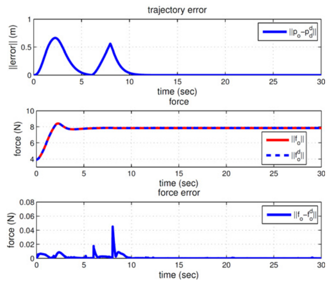
compliant object pushing



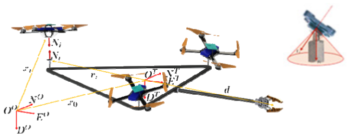


trajectory tracking with unknown added mass





SmQT System: Modeling



- Quadrotors (off-the-shelf) attached via spherical joints
- Multiple robots as distributed rotating thrusters
- Increased payload/dexterity with **over-actuation.**
- Control design under range limit of spherical joints.

- **Spherical joint limit constraints**

$$\boxed{p_i^T R_0^T R_i e_3 - \cos\phi_i \geq 0}$$

$p_i \in \Re^3, \phi_i \in \Re$: joint center axis and motion range
$\Gamma_i = R_0^T R_i e_3 \in \Re^3$: quadrotor thrust in fixed frame
$R_0, R_i \in SO(3)$: tool and quadrotor attitude

- **Quadrotor attitude dynamics**

$$\boxed{J_i\dot{\omega}_i = -S(\omega_i)J_i\omega_i + \tau_i}$$

decoupled from tool dynamics

$J_i \in \Re^{3\times3}$: inertia matrix
$\omega_i \in \Re^3$: angular velocity
$\tau_i \in \Re^3$: control torque

- **Tool dynamics**

control input

$$\boxed{M\dot{\xi} + C + G = U + F_e}$$

$M(R_0, R_i) \in \Re^{6\times6}$: inertia matrix
$C(R_0, \omega_0) \in \Re^6$: centrifugal/Coriolis term
$G(R_0) \in \Re^6$: gravity
$\omega_0 \in \Re^3$: tool angular velocity
$F_e \in \Re^6$: external force

- **Tool control input**

$$\boxed{U = \bar{R}B\Gamma}$$

quadrotor thrusts as control actions

$$\Gamma := \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \dots \\ \Gamma_n \end{bmatrix} = \begin{bmatrix} \lambda_1 R_0^T R_1 e_3 \\ \lambda_2 R_0^T R_2 e_3 \\ \dots \\ \lambda_n R_0^T R_n e_3 \end{bmatrix} \quad \bar{R} := -\begin{bmatrix} R_0 & 0 \\ 0 & I \end{bmatrix} \quad B := \begin{bmatrix} I & I & \dots & I \\ S(r_1) & S(r_2) & \dots & S(r_n) \end{bmatrix}$$

Control Allocation with Spherical Joints

- Finding control Γ_i while respecting the spherical joint constraints

$$\min_{\Gamma_1,\Gamma_2,\ldots,\Gamma_n\in\Re^3} \frac{1}{2}\Gamma^T\Gamma$$
$$\text{subject to} \quad B\Gamma = \bar{R}^{-1}U \quad \leftarrow \text{generate desired tool wrench}$$
$$p_i^T\Gamma_i \geq |\Gamma_i|\cos\phi_i \quad \leftarrow \text{respect the spherical joint constraint}$$

Generate desired tool wrench

"fully-actuated" iff $\mathrm{rank}(B) = 6$

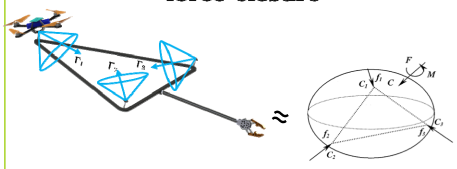
$$B\Gamma = \begin{bmatrix} I & I & I \\ S(r_1) & S(r_2) & S(r_3) \end{bmatrix} \begin{bmatrix} \lambda_1 R_0^T R_1 e_3 \\ \lambda_2 R_0^T R_2 e_3 \\ \lambda_3 R_0^T R_3 e_3 \end{bmatrix}$$
$$= \begin{bmatrix} I & 0 \\ S(r_1) & I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & S(r_2 - r_1) & S(r_3 - r_1) \end{bmatrix} \begin{bmatrix} I & I & I \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

$$\boxed{\mathrm{rank}(B) = 6 \leftrightarrow (r_2 - r_1) \times (r_3 - r_1) \neq 0}$$

Prop. 1: 6-DOF tool fully-actuated iff <u>at least three</u> quadrotors are used with their attaching points r_i <u>not collinear</u>.

Respect the spherical joint constraint

"force-closure"



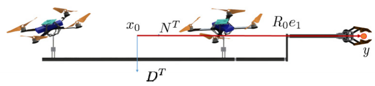
tool is in "force-closure" with Γ_i (Li at el. IEEE-TRA 2003)

Prop. 2: For fully-actuated tool system, <u>any desired tool wrench</u> can be generated under spherical limit, iff thrust actuations constitute <u>force-closure</u>.

S2QT System: Control Design

- **Control objective**

$$(y, R_o e_1) \rightarrow (y^d, \gamma_d)$$



- **Kinematics relation**

$$y = x_o + R_o d \rightarrow \dot{y} = \dot{x}_o + R_o S(\omega_o) d$$

consider as a virtual control

Assumptions and limitations:

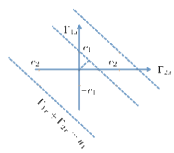
- Not fully-actuated, not in force-closure
- $d \times (r_2 - r_1) = 0$, $\tau_e \approx 0$
- Tool 5-DOF actuation

- **Lyapunov function**

$$V_1 = \frac{1}{2} e_y^T e_y + \frac{1}{2}\sum_{i=1}^{2} m_i \dot{e}_x^T \dot{e}_x + \frac{1}{2}\omega_0^T (J_0 - \sum_{i=1}^{2} m_i S^2(r_i))\omega_0 + k_R(1 - \gamma_d^T R_0 e_1)$$

enforce $y \rightarrow y_d$

enforce $(\omega_0, R_0 e_1) \rightarrow (0, \gamma_d)$



- **Control allocation**

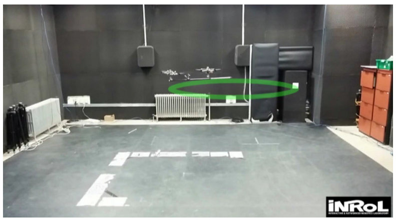
$$\min_{\Gamma_1,\Gamma_2\in\Re^3} \frac{1}{2}(\Gamma_1^T\Gamma_1 + \Gamma_2^T\Gamma_2)$$
$$\text{subject to} \begin{bmatrix} I & I \\ S(r_1) & S(r_2) \end{bmatrix} \begin{bmatrix} \lambda_1 R_0^T R_1 e_3 \\ \lambda_2 R_0^T R_2 e_3 \end{bmatrix} = \begin{bmatrix} F_d \\ M_d \end{bmatrix}$$
$$p_i^T\Gamma_i \geq |\Gamma_i|\cos\phi_i$$

Closed-form solution exists if

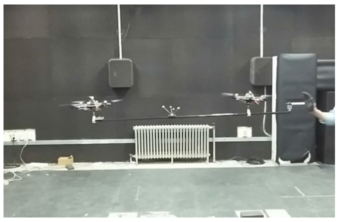
$$|F_{dx}| < c_1 + c_2$$
$$c_i = \frac{1 - \cos^2\phi_i}{\cos^2\phi_i}\Gamma_{iz}^2 - \Gamma_{iy}^2 > 0$$

$$\boxed{\Rightarrow (y, R_0 e_1) \rightarrow (y_d, \gamma_d) \text{ asymptotically}}$$

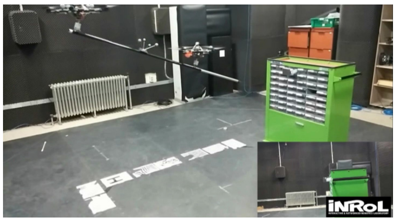
S2QT Preliminary Experiment



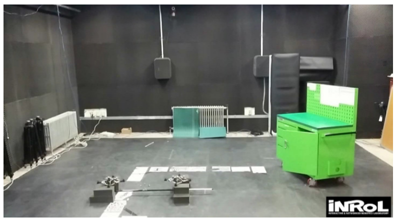
Trajectory tracking (error < 5cm)



Impedance control (contact force > 14N)



drawer pushing teleoperation



door closing teleoperation

Conclusion and Future Direction

- Control of physical interaction in/with multi-robot systems
  - Decentralized: large number, loose coherency, local interaction
  - Centralized: tight coherency, small number, central coordination
- Decentralized control of physical interaction
  - Multi-UAV teleoperation: interplay btw collective and local control
  - Multi-user haptics: high-level consensus + physical interaction
- Centralized control of physical interaction
  - Nonholonomic mobile manipulators: behavior decomposition
  - Multiple QM systems: complex dynamics and large-DOF
  - SmQT system: multi-robot system as distributed actuators
- Future directions
  - Almost fully-decentralized but still precise coherency?
  - Physical interaction control fused with algorithms?