Control of the Physical Interaction in/with Multi-Robot Systems

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Multi-Robot Physical Interaction

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Categorization

**Decentralized**
- Local control and communication
- Behavioral control, swarming, etc.
- Large number of robots
- Loose mechanical coherency

**Centralized**
- Central control and communication
- Precise mechanical coherency
- Relatively small number of robots
- Single robot or multiple robots more or less the same

Issues of Their Own

**Decentralized**
- Desired behavior only from local interaction
- Partial connectivity and latency
- Collision avoidance vs separation prevention
- Provable emergent behavior
- Consensus/synchronization on graph

**Centralized**
- Central communication and control expensive
- Precise and tight coordination
- High performance from real robots
- Real robots w/ complex dynamics, constraints, etc.
- Hybrid position/force control, behavior decomposition
Content

- Decentralized vs Centralized
- Decentralized control of physical interaction
  - Overview
  - Multi-UAV teleoperation
  - Multi-user haptic interaction
- Centralized control of physical interaction
  - Overview
  - Multiple mobile manipulators
  - Multiple quadrotor-manipulator systems
  - Spherically-connected multi-quadrotor (SmQT) systems
- Conclusion and future directions

Decentralized Physical Interaction Control

- Simple first-order consensus equation:
  \[ \dot{x}_i = -\sum_{j \in \mathcal{N}_i} w_{ij}(x_i - x_j) \]

where \( \mathcal{N}_i \in \mathcal{V} \) is information neighbor on graph \( G = (\mathcal{V}, \mathcal{E}, \mathcal{W}) \).

- Each robot abstracted by a simple dynamics (e.g., point mass in \( \mathbb{R}^3 \))
- System communication/control restricted by the topology of graph \( G \)
- Closed-loop dynamics:
  \[ \ddot{x} = -Lx, \quad x = [x_1, x_2, \ldots, x_n] \in \mathbb{R}^{3n} \]

where \( L \) is Laplacian matrix with \( \lambda_1(L) \geq 0 \) with 0 being simple iff \( G \) has a spanning tree (i.e., marginally stable).
Decentralized Physical Interaction Control

- Total control law = collective control + local control

\[ \dot{x}_i = u_i(t) - \sum_{j \in N_i} w_{ij}(x_i - x_j) \Rightarrow \dot{x} = -Lx + u \]

- \( u_i(t) \in \mathbb{R}^3 \): collective control for some agents to drive whole group (e.g., human command, virtual leader)

- \( w_{ij}(||x_i - x_j||) \): local control to maintain coherency while avoiding collision on graph \( G \)

- How to maintain local behavior (e.g., avoidance, coherency) even with unpredictable \( u_i(t) \)? (cf. string stability, Swaroop et al, TAC96)

- Collective control \( u_i(t) \) often cognitive, whereas local control \( w_{ij}(||x_i - x_j||) \) typically mechanical

Pseudo physical interaction \( \Leftrightarrow \) indirectly via motion control

Multi-UAV Teleoperation

- issues/challenges:
  - single user can manage only small-DOF
  - information-flow among UAVs should be distributed, yet, no collision/separation under arbitrary human tele-command

* semi-autonomous teleoperation
  = teleoperation + local autonomous control

1. UAV control layer (bksteping):
  - under-actuated UAV tracks its own kinematic virtual point (VP)

2. VP control layer:
  - NVPs as a deformable flying object on \( G \)
  - deforms to obstacles w/o VP-VP separation or VP-obstacle/VP-VP collisions

3. teleoperation layer:
  - PSPM for flexible/stable teleoperation
Distributed VP Control Layer

- render $N$ kinematic VPs as a $N$-nodes deformable flying object with artificial potentials distributed over connectivity graph $G$
- same architecture can be used for interaction with real objects

**kinematic VP**

\[
\dot{p}_i(t) := u_i^t + u_i^o
\]

**prop. 1:** Suppose \( \|u_i^t\| \leq \bar{u} \forall t \geq 0 \), and, if \( V(t) \geq M, \) \( \exists \) at least one VP, s.t.,

\[
\sum_{j \in \mathcal{N}_i} \frac{\partial \phi_i}{\partial p_i} + \sum_{r \in \mathcal{O}_i} \frac{\partial \phi_r}{\partial p_i} \geq \frac{\sqrt{N_i + \delta_{st}}}{2} \bar{u} \quad \delta_{st} = 1 \text{ if } s \in N_t; \quad \delta_{st} = 0 \text{ if } s \notin N_t
\]

Then, all VPs are stable (i.e., bounded $\dot{p}_i$); no VP-VP/VP-obstacle collisions; and no VP-VP separations.

\[
\frac{dV}{dt} = \sum_{i=1}^{N} \left( \sum_{j \in \mathcal{N}_i} \frac{\partial \phi_i}{\partial p_j} + \sum_{r \in \mathcal{O}_i} \frac{\partial \phi_r}{\partial p_j} \right) \dot{p}_i = \sum_{i=1}^{N} W_i^T(-W_i + u_i^o) \leq \sum_{i=1}^{N} (-||W_i||^2 + \delta_{st}||W_i||^2) \\
\leq -\left(||W_i|| - \frac{\delta_{st} \bar{u}}{2}\right)^2 + N_i \frac{\bar{u}^2}{4} \leq 0 \quad \rightarrow V(t) \leq M
\]

- only one VP needs to detect $V(t) \geq M$, w/ potential not exactly aligned
- stable for any bounded teleoperation command $u_i^t \Leftarrow$ guaranteed by PSPM
Experiments

- Distributed multi-UAV teleoperation: flying over obstacle
- Distributed multi-UAV teleoperation: adapt to narrow passage

* Distributed multi-UAV teleoperation: flying over obstacle w/o inter-UAV collision/separation on graph $G$ under arbitrary human command
* Distributed multi-UAV teleoperation: adapt to narrow passage w/o inter-UAV collision/separation on graph $G$ under arbitrary human command

- Haptic teleoperation of UAV
- Multi-modal semi-autonomous teleoperation of UAV/VG

- Open cap using UAV teleoperation with haptic feedback and communication delay
- Multi-modal semi-autonomous teleoperation crucial for complex real-world applications

* Joint work with Max Planck Institute for Biological Cybernetics, Tübingen, Germany

Multiuser Haptic Interaction

- Multiple haptic devices (i.e., robots) interconnected over graph $G$
- Each robot has "real" physical interaction with human
- Source of physical instability: human-device interaction, information delay
- Consensus for consistency (i.e., coordination) and passivity for stability
**Discrete-Time Passivity**

Suppose VO synchronization gains $B_i, K_i$ are set to be:

$$B_i \geq \sum_{j \in N_i} \frac{N_{ij} + N_{ji}}{2} \max_{k}[T_j(k)] K_{ij} \quad \forall \text{ users } i=1,...,N$$

Then, total p2p architecture is \textbf{N-port discrete-time passive}: \( \forall M \geq 0 \),

$$\sum_{k=0}^{\infty} \sum_{i=1}^{N} Y^T(k) f_i(k) T_i(k) \geq V(M + 1) - V(0) + \sum_{k=0}^{\infty} \sum_{i=1}^{N} \|v_i(k)\|^2 T_i(k)$$

- non-iterative passive integrator [DSCCo8]
- passive synchronization [ACC10]
- extend [Lee&SpongTRO06] to discrete domain
- robust stability for \textit{any} devices \& passive users
- not require specific kind/number of device/user
  \( \rightarrow \) portability/scalability for heterogeneous devices/users

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**Local Copy Synchronization**

If user forces $f_i(k) \rightarrow 0$ and VO damping $B$ is positive-definite, $v_i(k) \rightarrow 0$ and VO local copies will be \textbf{configuration-synchronized} s.t.

$$[\mathcal{P} + I_{N \times N} \otimes K_0] \left( x(k) - 1_N \otimes x_d \right) \rightarrow 0$$

- all the VO local copies' configurations $x(k) = [x_1(k); x_2(k); ...; x_N(k)] \in \mathbb{R}^{3N}$ converge to stable equilibria $x(k) \rightarrow \text{null}(\mathcal{P}) \cap \text{null}(I_N \otimes K)$
- VO synchronization guaranteed with $\text{null}(\mathcal{P}) = \{x_i = x_j = d, d \in \mathbb{R}^3\}$
  if $G$ is \textit{connected} [Huang&LeeACC10]
- $K_{\text{int}}$: VO internal shape
- $K_{\text{ext}}$: symmetry breaking
  e.g. if $K_{\text{ext}} = \alpha_i x_i \rightarrow \text{null}(I_N \otimes c_i, c \in \mathbb{R})$
Which (connected) network topology should we choose? 
→ **fastest mixing graph** \( G_{\text{opt}} \) from the set of all candidates graphs \( G_i \)

Information mixing model:

\[
p_i(k+1) = \left( \sum_{j \in N_i} K_{ij} \right)^{-1} \sum_{j \in N_i} K_{ij} p_j(k+1 - N_{ij})
\]

- user \( i \)'s information state
- normalization
- \( K_{ij} \): information mixing strength
- communication delay

**Experiments**

- multiuser haptic interaction: best graph topology
- multiuser haptic interaction: worst graph topology
- multiuser finger-based haptic interaction over the Internet
Centralized Physical Interaction Control

- Single robot as kinematic or dynamic system:
  \[ \dot{\mathbf{x}} = J(q)\dot{q}, \quad M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau + f \]

- If stack-up, multirobot system the same as single robot system:
  \[ \dot{x} = J(q)\dot{q}, \quad M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau + f \]
  \( w/ x = [x_1, x_2, ..., x_n], \quad q = [q_1, q_2, ..., q_n], \quad \tau = [\tau_1, \tau_2, ..., \tau_n], \quad f = [f_1, f_2, ..., f_n], \quad J = \text{diag}[J_1, J_2, ..., J_n], \quad M = \text{diag}[M_1, M_2, ..., M_n], \quad C = \text{diag}[C_1, C_2, ..., C_n] \)

- Null-space based control: with two tasks \( r_1 = f_1(q), \quad r_2 = f_2(q), \) \( r_1 \) of higher priority,
  \[ \dot{q} = J_1^T r_1 + (J_2 P_1)^+ [r_2 - J_2 \dot{q}_1] \]
  where \( P_1 \) is projection to null-space of \( J_1 \).

- Hybrid position/force control: with the task specified by holonomic constraint \( h(q) = c \) (or \( q = f(\phi), \dot{\phi} = J(\phi)\phi) \),
  \[ \tau = M(q)J(\phi)(\ddot{\phi}_4 - K_v \dot{\phi} - K_p \phi) + [C(q, \dot{q})J(\phi) + M(q)J(\phi)]\dot{\phi} \]
  \[ + g(q) - f + A^T(q)[\lambda - K_f \int (\lambda - \lambda_d)dt] \]

Centralized Physical Interaction Control

- Kinematic or dynamic modeling of multirobot system:
  \[ \dot{x} = J(q)\dot{q}, \quad M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau + f \]

- Null-space control, hybrid control \( \Rightarrow \) behavior decomposition

- Multirobot with centralized control \( \Rightarrow \) more or less same as single robot

- Then, what is so unique about multirobot system?
  - Large-DOF \( \Rightarrow \) difficult to control all the same
  - Large-DOF \( \Rightarrow \) richer behavioral decomposition possible/demanded
  - As compared to a single robot, real multirobot would likely have
    * More complex dynamics (e.g., platform-arm system)
    * More abundance of constraints (e.g., wheels, under-actuation)
    * Heterogeneity among the robots

- Behavior decomposition \( w/ \) complex dynamics, constraint, heterogeneity
**Multirobot Fixture-Less Grasping**

- total-DOF = 15
- three behaviors:
  1) **grasping**
  2) grasped object **maneuver**
  3) **internal** motion (e.g., avoidance, reconfiguration)

→ decomposition into these three behaviors even with nonholonomic constraints?

**Behavior Decomposition and Control**

* hierarchical control
  = simultaneous/separate control of each behavioral mode autonomously or teleoperatedly

\[ \dot{q} = \lambda_k \xi^k + (u_x \xi_x + u_y \xi_y) + \sum_{i=1}^{3} \alpha_i \xi_i^i + \sum_{i=1}^{3} \beta_i \xi_i^i + \sum_{i=1}^{3} 0 \cdot \xi_i^i \]

- grasping
- object maneuvering
- internal motion (avoidance, reconfiguration)

**Equilibrium**

\[ \phi_1 = \frac{1}{2} \sin \phi_1 \]

\[ \phi_1 > 0, \phi_1 < 0 \]

<table>
<thead>
<tr>
<th>( \xi_1 )</th>
<th>( \phi_1 )</th>
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<tr>
<td>( &gt; 0 )</td>
<td>( \phi \rightarrow \pi )</td>
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<tr>
<td>( &lt; 0 )</td>
<td>( \phi \rightarrow \pi )</td>
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**Required level of intelligence**

- simple autonomous
- cognitive
- autonomous, yet rich (or teleoperation?)
**NPD Expression**

\[
\dot{q} = \lambda_0 \dot{x} + (u_x \dot{x} + u_y \dot{y}) + \sum_{i=1}^{3} \alpha_i \dot{x}_i + \sum_{i=1}^{3} \beta_i \dot{y}_i + \sum_{k=1}^{3} 0 \cdot \dot{z}_k 
\]

- **grasp regulation** \( \lambda_0 = -\frac{1}{2} \sqrt{\frac{1}{2} \frac{k}{m}} \)
- **object maneuvering:** teleoperation
- **internal motion:** autonomous (rich)
- **grasping:** simple autonomous

**Simulation**

- autonomous obstacle avoidance using combination of maneuver mode and internal dynamics mode
- rigid grasping enforced with no grip-holding fixture

- autonomous grasping control
- object maneuver haptic teleoperation completely decoupled from grasping behaviors

- object teleoperation with interaction force feedback
- rigid grasping maintained regardless of object interaction
Quadrotor-Manipulator System

- Dexterous aerial manipulation using quadrotor-arm system is promising.
- Quadrotor-arm dynamic coupling → dynamics formulation necessary.
- QM system dynamics very complex with 2-DOF under-actuation.
- Slow/imprecise quadrotor flying + fast/precise manipulator control.
- Reveal fundamental underlying dynamics structure; exploit it for control.

Configuration-Space Decomposition

- Tangent space decomposition
  \( q := [p; \phi; \theta] \in \mathbb{R}^n \quad r := [\phi; \theta] \in \mathbb{R}^{n+3} \)
  
  Choose \( h(q) := r \)

  \[ \dot{q} = \begin{bmatrix} \Delta_T & \Delta_L \end{bmatrix} \nu = \begin{bmatrix} I_3 & -\frac{1}{m_L}M_{pr}(r) \\ 0 & I_{n-3} \end{bmatrix} \begin{bmatrix} \dot{p}_L \\ \dot{r} \end{bmatrix} \]

- Passive decomposition [ICRA14]

  \[ M(r)\ddot{q} + C(r, \dot{r})\dot{q} + g(q) = \tau + f \]

  \[ m_L\ddot{p}_L + g_L = \tau_L = \lambda R_0(\psi) \]

  \[ M_E(r)\ddot{r} + C_E(r, \dot{r})\dot{r} = \tau_E \]

- QM dynamics = centroid \( p_L \)-dynamics + internal rotation \( r \)-dynamics
- \( p_L \)-dynamics = standard under-actuated quadrotor dynamics.
- \( r \)-dynamics = standard fully-actuated robot-arm dynamics.
- No inertial/gravity coupling w/ gravity only in \( p_L \)-dynamics.

⇒ Composed of completely-decoupled \( p_L \) and \( r \) dynamics on their manifolds.
⇒ Generalize rigid-body dynamics in \( \text{SE}(3) \) to floating multi-link systems.
⇒ Applicable to any vehicle-manipulator systems (e.g., ROV, space robot).
Coarse-Fine QM-System Control

- Backstepping end-effector tracking control with redundancy
  \[ \tau_L(\lambda, \phi) + m_L B(r) M_{E^{-1}}(r) \tau_E = -\gamma e_p - \alpha e_L + g_L + \eta(r) + m_L [p_L - \lambda e_p - \frac{dB}{dr} r] \]

  Control redundancy: cooperative control

- Slow \( p_L \)-dynamics w/ under-actuated control \( \tau^L_2(\lambda, \phi_2) \) => coarse control
  \[ \tau^L_2(\lambda, \phi_2) : = \text{LPF} \left[ -\gamma e_p - \alpha e_L + g_L + \eta(r) + m_L [p_L^d - k_e e_p - \frac{\partial B}{\partial r} r] \right] - m_L \dot{\lambda} \zeta(r) \]

- Fast \( r \)-dynamics w/ fully-actuated control \( \tau_E \) => fine control
  \[ m_L B M_{E^{-1}} \tau_E = -\gamma e_p - \alpha e_L + g_L + \eta(r) + m_L [p_L^d - k_e e_p - \frac{\partial B}{\partial r} r] - \tau_L(\lambda, \phi) \]

- Trajectory tracking \((e_p, e_L) \to 0\) guaranteed even with \( \tau_L \neq \tau^L_2 \).

- Latitude for \( \tau^L_2 \): arm sub-task \( \zeta(r) \Rightarrow m_L B \dot{r} \rightarrow \zeta(r) \) (e.g., impedance)

- Singularity avoidance by adapting cut-off frequency \( \omega_c(\sigma(\theta)) \) of LPF.

- Redundant manipulator control \( \tau_E \in \text{nullspace}(BM_{E^{-1}}) \):
  - Quadrotor attitude control to align \( \tau_L \rightarrow \tau^L_2 \)
  - Optimal arm posture, collision/obstacle avoidance.

QM System Control Examples

end-effector trajectory tracking

coarse-fine control

tracking-obstacle avoidance

admittance force control
Hierarchical Cooperative Control Framework

- **Object behavior design**: computes required object wrench to produce user-specific target behaviors (e.g., trajectory tracking, compliant interaction, etc.)

- **Optimal cooperative force distribution**: optimally assign contact force for each QM system to achieve the target behavior under uni-lateral/friction contact constraint.

- **Individual QM system control**: admittance force control with unknown object stiffness with coarse-fine redundancy resolution.

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Object Behavior Design Layer

- **Object behavior design**: compute desired object wrench to produce user-specific behaviors (e.g., trajectory tracking, compliant interaction).

- **Object rigid-body dynamics**:

  \[
  m_o \ddot{x}_o + m_o g = \mathbf{f}_o + \mathbf{f}_{ext} \\
  I_o \ddot{\omega}_o + \omega_o \times I_o \omega_o = \tau_o + \tau_{ext} \\
  \]

- **Compute object wrench \( \mathbf{F}_o = [f_o, \tau_o] \)**, e.g., for impedance control:

  \[
  f_o = m_o g + m_o \dot{\omega}_o^2 + D(\dot{\theta}_o^d - \theta_o) + K(\theta_o^d - \theta_o) \\
  \tau_o = -\gamma \omega_o - \alpha [R_o - \dot{R}_o^d] \\
  \]
**Optimal Cooperative Force Distribution Layer**

- **Optimal cooperative force distribution**: optimally assign contact force of each QM system to achieve the target behavior w/o dropping the object (i.e., friction cone constraint).

- **Jacobian relation**:

  \[ F_o = J_o \tilde{f}_o, \quad J_o \in \mathbb{R}^{d \times 3N} \]

  where \( \tilde{f}_o = [f_{o,1}; f_{o,2}; \ldots; f_{o,N}] \in \mathbb{R}^{3N} \) is all QM systems’ EF forces.

- **\( J_o \)** fat matrix with friction cone constraints \( \Rightarrow \) constrained optimization:

  \[
  \min_{f_{zn}} \quad \alpha_1 f_n^T f_n + \alpha_2 \tilde{f}_n^T f_n \\
  \text{subj. to} \quad F_o = J_o N f_n + J_o T f_s \\
  \sqrt{f_{s,1}^2 + f_{s,2}^2} \geq \mu f_{o,i}, \quad i = 1, 2, \ldots, N
  \]

**Admittance Force Control Layer**

- **Imprecise position control of quadrator \( \Rightarrow \) force control.**

- **Robot-arm likely not backdrivable \( \Rightarrow \) admittance control.**

- **Local object deformation model with unknown stiffness matrix \( K_o \):**

  \[ f_{o,i} = -K_o (p_{o,i} - p_{o,i}) \]

- **Target position evolution:**

  \[ \dot{p}_{o,i}^d := k_1 (f_{o,i} - f_{o,i}^d) + k_2 \int (f_{o,i} - f_{o,i}^d) dt + \dot{p}_{o,i} \]

- **Lyapunov function for admittance-like force control**

  \[ V := \frac{1}{2} e_f^T e_f + \frac{1}{2} e_{\dot{f}}^T k_2 K_o e_f + e_f^T e_f + \frac{1}{2} e_{\dot{f}}^T k_0 e_{\dot{f}} \]

- **Control generation equation**

  \[
  \tau_L (\lambda, \phi) + m_L B(r) M^{-1}(r) \tau_E = -f_L m_L B M^{-1} I E + g_L \\
  + \eta(r) + m_L [\dot{p}_{o,i} - \beta \dot{e}_p] - \gamma (e_f + \epsilon \int e_f dt) - \frac{dB}{dt} \]

- (\( e_p, e_f, e_{\dot{f}} \)) ultimately bounded even w/ unknown \( K_o \) & \( \tau_L \neq 0 \).

- Slower-quadrator/faster-manipulator can be incorporated.

- Force sensing or estimator for force feedback.
Multiple Cooperative QM-System

SmQT System: Modeling
- Quadrors (off-the-shelf) attached via spherical joints
- Multiple robots as distributed rotating thrusters
- Increased payload/dexterity with over-actuation.
- Control design under range limit of spherical joints.

- Spherical joint limit constraints
  \[ p_i^2 R_i^2 R_i^c \leq \cos \phi_i \geq 0 \]
  \( p_i, R_i, \phi_i \in \mathbb{R} \): joint center axis and motion range
  \( R_i = R_i^c R_i^c \in \mathbb{R}^3 \): quadrotor thrust in fixed frame
  \( R_i, R_i^c \in SO(3) \): tool and quadrotor attitude

- Tool dynamics
  \[ M_i^\xi + C + G = U + F_e \]
  \( M_i^\xi \in \mathbb{R}^{6 \times 6} \): inertia matrix
  \( C(T_i, \tilde{\omega}) \in \mathbb{R}^{6\times6} \): centrifugal/Coriolis term
  \( G(T_i, \tilde{\omega}) \in \mathbb{R}^{6\times1} \): gravity
  \( F_e \in \mathbb{R}^{6\times1} \): external force
  \( \tilde{\omega} \in \mathbb{R}^3 \): tool angular velocity

- Tool control input
  \[ U = \tilde{R} B \Gamma \]
  \( \Gamma := \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \ldots \\ \Gamma_n \end{bmatrix} \)
  \( \begin{bmatrix} \lambda_1 R_1 R_1 R_1 \\ \lambda_2 R_2 R_2 R_2 \\ \lambda_n R_n R_n R_n \end{bmatrix} \)
  \( \tilde{R} := \begin{bmatrix} R_1 & 0 & \ldots & 0 \\ 0 & I & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & R_n \end{bmatrix} \)
  \( B := \begin{bmatrix} I \\ S(r_1) \\ S(r_2) \\ \ldots \\ S(r_n) \end{bmatrix} \)
Control Allocation with Spherical Joints

- Finding control $\Gamma_i$ while respecting the spherical joint constraints

$$\min_{\Gamma_1, \Gamma_2, \ldots, \Gamma_n} \frac{1}{2} \Gamma^T \Gamma$$
subject to $B \Gamma = R^{-1} U$
$$p_i^T \Gamma_i \geq |\Gamma_i| \cos \phi_i$$

Generate desired tool wrench

"fully-actuated" iff $\text{rank}(B) = 0$

$$B^T = \begin{bmatrix} I & I \\ S(r_1) & S(r_2) \end{bmatrix} \begin{bmatrix} \lambda_1 R_1 R e_1 \\ \lambda_2 R_2 R e_2 \end{bmatrix}$$
$$= \begin{bmatrix} I \\ S(r_1) \end{bmatrix} \begin{bmatrix} 0 & I \\ S(r_2) & S(r_2 - r_1) \end{bmatrix} \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}$$

$\text{rank}(B) = 6$ $\implies (r_2 - r_1) \times (r_3 - r_1) \neq 0$

Prop. 1: 6-DOF tool fully-actuated iff at least three quadrators are used with their attaching points $r_i$ not collinear.

Respect the spherical joint constraint

"force-closure"

tool is in "force-closure" with $\Gamma$, (Li et al. IEEE-TRA 2003)

S2QT System: Control Design

- Control objective

$$\langle y, R e_1 \rangle \rightarrow \langle y^d, \gamma_d \rangle$$

- Kinematics relation

$$y = x_a + R_e d \rightarrow \dot{y} = \dot{x}_a + R_e \omega \omega d$$

- Lyapunov function

$$V_1 = \frac{1}{2} e_T e_T + \frac{1}{2} \sum_{i=1}^{2} m_i \dot{e}_i \dot{e}_i + \frac{1}{2} \omega^2 (J_D - \sum_{i=1}^{2} m_i S^2(r_i)) \omega \omega + k_n (1 - \gamma^2_R R e_1)$$

- Control allocation

$$\min_{\Gamma_1, \Gamma_2, \ldots, \Gamma_n} \frac{1}{2} \Gamma^T \Gamma$$
subject to $\begin{bmatrix} I & I \\ S(r_1) & S(r_2) \end{bmatrix} \begin{bmatrix} \lambda_1 R_1 R e_1 \\ \lambda_2 R_2 R e_2 \end{bmatrix} = \begin{bmatrix} P_{du} \\ M_d \end{bmatrix}$
$$|P_{du} - c_1 + c_2 | < c_3$$
$$c_4 = \frac{1 - \cos^2 \phi_i}{\cos^2 \phi_i} 1_{x^2} - 1_{y^2} > 0$$

Closed-form solution exists if

$$\Rightarrow (y, R e_1) \rightarrow (y^d, \gamma_d)$$ asymptotically
S2QT Preliminary Experiment

Trajectory tracking (error < 5cm)  
Impedance control (contact force > 14N)

drawer pushing teleoperation  
door closing teleoperation

Conclusion and Future Direction

- Control of physical interaction in/with multi-robot systems
  - Decentralized: large number, loose coherency, local interaction
  - Centralized: tight coherency, small number, central coordination
- Decentralized control of physical interaction
  - Multi-UAV teleoperation: interplay between collective and local control
  - Multi-user haptics: high-level consensus + physical interaction
- Centralized control of physical interaction
  - Nonholonomic mobile manipulators: behavior decomposition
  - Multiple QM systems: complex dynamics and large-DOF
  - SmQI system: multi-robot system as distributed actuators
- Future directions
  - Almost fully-decentralized but still precise coherency?
  - Physical interaction control fused with algorithms?